

Noncommutative Space-Time

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Abstract

Quantum field theories based on noncommutative space-time (NCQFT) have been extensively studied recently. However no NCQFT model, which can uniquely describe the strong and electroweak interactions, has been constructed. This prevents consistent and systematic study of noncommutative space-time. In this work we construct a NCQFT model based on the trification gauge group $SU(3)_C \times SU(3)_L \times SU(3)_R$. A unique feature of this model, that all matter fields (fermions and Higgses) are assigned to (anti-)fundamental representations of the factor $SU(3)$ groups, allows us to construct a NCQFT model for strong and electroweak interactions and their unification without ambiguities. This model provides an example which allows consistent and systematic study of noncommutative space-time phenomenology. We also comment on some related issues regarding extensions to E_6 and $U(3)_C \times U(3)_L \times U(3)_R$ models.

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Noncommutative quantum field theory (NCQFT), based on modification of the space-time commutation relations, provides an alternative to the ordinary quantum field theory. A simple way to modify the space-time properties is to change the usual space-time coordinate x to noncommutative coordinate \hat{X} such that [1]

$$[\hat{X}^\mu, \hat{X}^\nu] = i\theta^{\mu\nu}, \quad (1)$$

where $\theta^{\mu\nu}$ is a real anti-symmetric matrix. We will consider the case where $\theta^{\mu\nu}$ is a constant and commutes with \hat{X}^μ .

NCQFT based on the above commutation relation can be easily studied using the Weyl-Moyal correspondence replacing the product of two fields $A(\hat{X})$ and $B(\hat{X})$ with noncommutative coordinates by product of the same fields but ordinary coordinate x through the star “*” product,

$$A(\hat{X})B(\hat{X}) \rightarrow A(x) * B(x) = \text{Exp}[i\frac{1}{2}\theta^{\mu\nu}\partial_{x,\mu}\partial_{y,\nu}]A(x)B(y)|_{x=y}. \quad (2)$$

Properties related to NCQFT have been studied extensively recently [2–11,13,14]. NCQFT for a pure $U(1)$ group is easy to study. Related phenomenology have been studied recently [2]. But it is more complicated for non-abelian groups. Due to the “*” product nature, there are fundamental differences between ordinary and noncommutative gauge theories and cause many difficulties to construct a unique and consistent model for strong and electroweak interactions based on $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge group [3–11,13,14]. One of the main problems is that $SU(N)$ group can not be simply gauged with “*” product as will be explain in the following. Another problem is that, naively, the charges of any $U(1)$ gauge group with “*” product are quantized to only three possible values, 1, 0, -1 which cannot accommodate all the hypercharges for matter fields in the SM.

In this work we construct a NCQFT model based on the trinification gauge group $SU(3)_C \times SU(3)_L \times SU(3)_R$. We show that NCQFT model for strong and electroweak interactions and their unification can be consistently constructed. This model therefore provides an example which allows a consistent and systematic study of noncommutative space-time phenomenology.

With noncommutative space-time there are modifications to the fields compared with the ordinary ones. We indicate the fields in NCQFT with a hat and the ordinary ones without hat. The definition of gauge transformation $\hat{\alpha}$ of a gauge field \hat{A}_μ for a $SU(N)$ is similar to the ordinary one but with usual product replaced by the “*” product. For example

$$\delta_\alpha \hat{\phi} = i\hat{\alpha} * \hat{\phi}, \quad (3)$$

where $\hat{\phi}$ is a fundamental representation of $SU(N)$. We use the notation $\hat{A}_\mu = \hat{A}_\mu^a T^a$, $\hat{\alpha} = \alpha^a T^a$ with T^a being the $SU(N)$ generator normalized as $Tr(T^a T^b) = \delta^{ab}/2$.

Due to the noncommutativity of the space-time, two consecutive local transformations $\hat{\alpha}$ and $\hat{\beta}$ of the type in the above,

$$(\delta_\alpha \delta_\beta - \delta_\beta \delta_\alpha) = (\hat{\alpha} * \hat{\beta} - \hat{\beta} * \hat{\alpha}), \quad (4)$$

cannot be reduced to the matrix commutator of the generators of the Lie algebra due to the noncommutativity of the space-time. They have to be in the enveloping algebra

$$\hat{\alpha} = \alpha + \alpha_{ab}^1 : T^a T^b : + \dots + \alpha_{a_1 \dots a_n}^{n-1} : T^{a_1} \dots T^{a_n} : + \dots \quad (5)$$

where $: T^{a_1} \dots T^{a_n} :$ is totally symmetric under exchange of a_i . This poses a difficulty in constructing non-abelian $SU(N)$ gauge theories [3].

Seiberg and Witten have shown [5] that the fields defined in noncommutative coordinate can be mapped on to the ordinary fields, the Seiberg-Witten mapping. In Ref. [6] it was shown that this mapping actually can be applied to the “*” product with any gauge groups. It is possible to study non-abelian gauge group theories. Using the above enveloping algebra, one can obtain the noncommutative fields in terms of the ordinary fields with corrections in powers of the noncommutative parameter, $\theta^{\mu\nu}$, order by order. To the first order in $\theta^{\mu\nu}$ noncommutative fields can be expressed as

$$\begin{aligned} \hat{\alpha} &= \alpha + \frac{1}{4} \theta^{\mu\nu} \{ \partial_\mu \alpha, A_\nu \} + c \theta^{\mu\nu} [\partial_\mu \alpha, A_\nu], \\ A_\mu &= -\frac{1}{4} \theta^{\alpha\beta} \{ A_\alpha, \partial_\beta A_\mu + F_{\beta\mu} \} + c \theta^{\alpha\beta} ([A_\alpha, \partial_\mu A_\beta] + i[A_\alpha A_\beta, A_\mu]), \\ \hat{\phi} &= a \theta^{\mu\nu} F_{\mu\nu} \phi - \frac{1}{2} \theta^{\mu\nu} A_\mu \partial_\nu \phi + i \left(\frac{1}{4} + c \right) \theta^{\mu\nu} A_\mu A_\nu \phi, \end{aligned} \quad (6)$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig_N[A_\mu, A_\nu]$. The term proportional to a can be absorbed into the redefinition of the matter field ϕ . The parameter c can not be removed by redefinition of the gauge field. It must be a purely imaginary number from the requirement that the gauge field be self conjugate.

Using the above noncommutative fields, one can construct a gauge theory for $SU(N)$ group. The action S of a $SU(N)$ NCQFT, to the leading order in θ , is given by [6,7],

$$\begin{aligned} S &= \int L d^4x, \\ L &= -\frac{1}{2}Tr(F_{\mu\nu}F^{\mu\nu}) + \frac{1}{4}g_N\theta^{\mu\nu}Tr(F_{\mu\nu}F_{\rho\sigma}F^{\rho\sigma} - 4F_{\mu\rho}F_{\nu\sigma}F^{\rho\sigma}) \\ &\quad + \bar{\phi}(i\gamma^\mu D_\mu - m)\phi - \frac{1}{4}\theta^{\alpha\beta}\bar{\phi}F_{\alpha\beta}(i\gamma^\mu D_\mu - m)\phi - \frac{1}{2}\theta^{\alpha\beta}\bar{\phi}\gamma^\mu F_{\mu\alpha}iD_\beta\phi, \end{aligned} \quad (7)$$

where $D_\mu = \partial_\mu - ig_N T^a A_\mu^a$. We note that the parameter c does not appear in the Lagrangian. The Lagrangian is uniquely determined to order θ . We will therefore work with a simple choice $c = 0$ from now on. In the above, if ϕ is a chiral field, $m = 0$.

To obtain a theory which can describe the strong and electroweak interactions such as the Standard Model (SM), one also needs to solve the $U(1)$ charge quantization problem, namely only three possible values, 1, 0, -1 for $U(1)$ charges, as mentioned earlier. It has been shown that this difficulty can also be overcome with the use of the Seiberg-Witten mapping [5].

To solve the $U(1)$ charge quantization problem, one associates each charge $gq^{(n)}$ of the n th matter field a gauge field $\hat{A}_\mu^{(n)}$ [8]. In the commutative limit, $\theta^{\mu\nu} \rightarrow 0$, $\hat{A}_\mu^{(n)}$ becomes the single gauge field A_μ of the ordinary commuting space-time $U(1)$ gauge theory. But at non-zero orders in $\theta^{\mu\nu}$, $\hat{A}_\mu^{(n)}$ receives corrections [8]. In doing so, the kinetic energy of the gauge boson will, however, be affected. Depending on how the kinetic energy is defined (weight over different field strength of $\hat{A}_\mu^{(n)}$), the resulting kinetic energy will be different even though the proper normalization to obtain the correct kinetic energy in the commutative limit is imposed [8].

In the SM there are six different matter field multiplets for each generation, i.e. $u_R, d_R, (u, d)_L, e_R, (\nu, e)_L$ and (H^0, H^-) , a priori one can choose a different g_i for each of them.

After identifying three combinations with the usual g_3 , g_2 and g_1 couplings for the SM gauge groups, there is still a freedom to choose different gauge boson self interaction couplings at non-zero orders in $\theta^{\mu\nu}$. This leads to ambiguities in self interactions of gauge bosons when non-zero order terms in $\theta^{\mu\nu}$ are included [8]. This problem needs to be resolved. A way to solve this problem is to have a theory without the use of $U(1)$ factor group.

There are many groups without $U(1)$ factor group which contain the SM gauge group and may be used to describe the strong and electroweak interactions. However not all of them can be easily extended to a full NCQFT using the formulation described above. For example one can easily obtain unique gauge boson self interactions in $SU(5)$ theory [9]. But the matter fields require more than one representations 5 and 10 which causes additional complications [10] and will reintroduce the uniqueness problem for kinetic energy. One way of obtaining a consistent NCQFT is to have a theory in which all matter and Higgs fields are in the same representation such that once Seiberg-Witten mapping is used to solve the problem of gauging a $SU(N)$ group, there is no problem with unique determination of kinetic energy. To this end we propose to use the trinification gauge group [15], $SU(3)_C \times SU(3)_L \times SU(3)_R$ with a Z_3 symmetry.

This theory leads to unification of strong and electroweak interactions. An important feature of this theory is that the matter and Higgs fields are assigned to (anti-)fundamental representations of the factor $SU(3)$ groups and therefore the formalism described earlier can be readily used.

In the trinification model, the gauge fields are in the adjoint representation,

$$24 = A^C + A^L + A^R = (8, 1, 1) + (1, 8, 1) + (1, 1, 8), \quad (8)$$

which contains 24 gauge bosons. A^C contains the color gluon bosons, a linear combination of component fields of A^L and A^R forms the $U(1)_Y$ gauge boson, and A^L contains the $SU(2)_L$ gauge bosons. The rest are integrally charged heavy gauge bosons which do not mediate proton decays [15].

Each generation of fermions is assigned to a 27,

$$\psi = \psi^{LR} + \psi^{RC} + \psi^{CL} = (1, 3, \bar{3}) + (\bar{3}, 1, 3) + (3, \bar{3}, 1),$$

$$\psi^{LR} = \begin{pmatrix} E^0 & E^- & e^- \\ E^+ & \bar{E}^0 & \nu \\ e^+ & N_1 & N_2 \end{pmatrix}, \quad \psi^{RC} = \begin{pmatrix} \bar{u}_1 & \bar{u}_2 & \bar{u}_3 \\ \bar{d}_1 & \bar{d}_2 & \bar{d}_3 \\ \bar{B}_2 & \bar{B}_2 & \bar{B}_2 \end{pmatrix}, \quad \psi^{CL} = \begin{pmatrix} u_1 & d_1 & B_1 \\ u_2 & d_2 & B_2 \\ u_3 & d_3 & B_3 \end{pmatrix}. \quad (9)$$

In the above we have written the fermions in left-handed chiral fields. The B field is a heavy particle.

The Higgs fields which break the trinification to the SM gauge group are also assigned to 27 representations. In order to have correct mass patterns, at least two 27 Higgs representations are needed [15]. We indicate them by

$$\phi_i = \phi_i^{LR} + \phi_i^{RC} + \phi_i^{CL} = (1, 3, \bar{3})_i + (\bar{3}, 1, 3)_i + (3, \bar{3}, 1)_i. \quad (10)$$

The Z_3 symmetry operates in the following way. If (C, L, R) is a representation under the $SU(3)_C \times SU(3)_L \times SU(3)_R$, the effect of Z_3 is to symmetrize it to

$$Z_3(C, L, R) = (C, L, R) + (R, C, L) + (L, R, C). \quad (11)$$

Requirement of the Lagrangian to be invariant under Z_3 relates the gauge couplings $g^{C,L,R}$ of the gauge groups to be equal, $g^C = g^L = g^R = g^U$, at a scale which is the unification scale of the model.

The vacuum expectation values of the Higgs scalars break the symmetry to $SU(3)_C \times SU(2)_L \times U(1)_Y$ have the following form

$$\langle \phi_1^{LR} \rangle = \begin{pmatrix} \hat{0} & 0 & 0 \\ 0 & \hat{0} & 0 \\ 0 & 0 & v_1 \end{pmatrix}, \quad \langle \phi_2^{LR} \rangle = \begin{pmatrix} \hat{0} & 0 & 0 \\ 0 & \hat{0} & \hat{0} \\ 0 & v_2 & \hat{0} \end{pmatrix}. \quad (12)$$

Non-zero values of $v_{1,2}$ break the symmetry to the SM group. The scale of $v_{1,2}$ are at the unification scale. At this stage 12 of the gauge bosons, and B , E_i and N_i particles receive masses. They are therefore very heavy. The entries indicated by $\hat{0}$ can develop VEV's of order m_W . These VEV's break the SM group to $SU(3)_C \times U(1)_{em}$ and provide masses for the ordinary quarks and leptons.

In this model $\sin^2 \theta_W = 3/8$ at the unification scale. Using the present electroweak precision test data for $\sin^2 \theta_W$, α_s and α_{em} at the Z mass pole, the unification scale is determined to be 10^{14} GeV [16]. A unification scale as low as 10^{14} GeV in a $SU(5)$ theory, for example, would induce rapid proton decays and be ruled out. However, as have been mentioned previously that in the trinification theory, gauge bosons do not mediate proton decays. Therefore the theory would not have problem with proton decays. Mediation of heavy Higgs particles can produce proton decays [15]. However in this case, there are many free parameters in the Yukawa and the Higgs potential couplings to make the theory to be consistent with data [15].

It is clear that the trinification model can provides an easy framework for building a phenomenologically acceptable and consistent NCQFT model for strong and electroweak interactions. To the first order in $\theta^{\mu\nu}$, the noncommutative gauge fields are the same form for the gauge fields as in eq. (6). The fermion and Higgs fields are in the same representation and are all (anti-)fundamental representations ϕ of the type $(3, \bar{3})$ under the subgroups $SU(3) \times SU(3)$. The noncommutative fields expressed in the ordinary fields are given by

$$\hat{\phi} = \phi - \frac{1}{2}\theta^{\mu\nu}(A_\mu\partial_\nu\phi - \frac{i}{2}A_\mu A_\nu\phi + \partial_\nu\phi A'_\mu + \frac{i}{2}\phi A'_\nu A'_\mu), \quad (13)$$

where A_μ and A'_μ are the gauge fields of the first $SU(3)$ and the second $SU(3)$ gauge groups, respectively.

The first order in $\theta^{\mu\nu}$ corrections to the Lagrangian \tilde{L} for gauge and fermion kinetic energy terms are given by

$$\begin{aligned} \tilde{L} = & \left[\frac{1}{4}g^C\theta^{\mu\nu}Tr(F_{\mu\nu}^C F_{\alpha\beta}^C F^{C\alpha\beta} - 4F_{\alpha\mu}^C F_{\beta\nu}^C F^{C\alpha\beta}) + (C \rightarrow L) + (C \rightarrow R) \right] \\ & - \left[\frac{i}{4}\theta^{\mu\nu}Tr(\bar{\psi}^{LR} F_{\mu\nu}^L \gamma^\alpha D_\alpha \psi^{LR} + F_{\mu\nu}^R \bar{\psi}^{LR} \gamma^\alpha D_\alpha \psi^{LR} \right. \\ & \left. + 2\bar{\psi}^{LR} F_{\alpha\mu}^L \gamma^\alpha D_\nu \psi^{LR} + 2F_{\alpha\mu}^R \bar{\psi}^{LR} \gamma^\alpha D_\nu \psi^{LR}) + (LR \rightarrow LC) + (LR \rightarrow CR) \right], \end{aligned} \quad (14)$$

where $D_\mu \psi^{LR} = \partial_\mu \psi^{LR} - ig_L A_\mu^L \psi^{LR} + ig_R \phi^{LR} A_\mu^R$.

The above Lagrangian uniquely determine interactions due to noncommutative space-time correction to the first order in $\theta^{\mu\nu}$ without the problems pointed out earlier. We

emphasis that although the resulting theory at low energies appears to have $U(1)$ factor group(s), the corresponding gauge self-interactions are fixed because of the choice of trification group which dictates how gauge bosons interact. From the above Lagrangian one can easily study new interactions due to noncommutative space-time and test the model by experimental data. For illustration, we present the neutral gauge boson self interactions and its interactions with the SM fermions. Expanding the above Lagrangian we obtain

$$\begin{aligned}
L_{int} = & \frac{1}{4}g^C\theta^{\mu\nu}Tr(G_{\mu\nu}G_{\alpha\beta}G^{\alpha\beta} - 4G_{\alpha\mu}G_{\beta\nu}G^{\alpha\beta}) \\
& + \frac{1}{16}\theta^{\mu\nu}g_Y[c_W(\frac{7}{15}c_W^2 + s_W^2)(F_{\mu\nu}F_{\alpha\beta}F^{\alpha\beta} - 4F_{\alpha\mu}F_{\beta\nu}F^{\alpha\beta}) \\
& - s_W(\frac{7}{15}s_W^2 + c_W^2)(Z_{\mu\nu}Z_{\alpha\beta}Z^{\alpha\beta} - 4Z_{\alpha\mu}Z_{\beta\nu}Z^{\alpha\beta}) \\
& + c_W(c_W^2 - \frac{23}{15}s_W^2)(F_{\mu\nu}Z_{\alpha\beta}Z^{\alpha\beta} + 2Z_{\mu\nu}Z_{\alpha\beta}F^{\alpha\beta} - 4(Z_{\alpha\mu}Z_{\beta\nu}F^{\alpha\beta} + 2F_{\alpha\mu}Z_{\beta\nu}Z^{\alpha\beta})) \\
& - s_W(s_W^2 - \frac{23}{15}c_W^2)(Z_{\mu\nu}F_{\alpha\beta}F^{\alpha\beta} + 2F_{\mu\nu}Z_{\alpha\beta}F^{\alpha\beta} - 4(F_{\alpha\mu}F_{\beta\nu}Z^{\alpha\beta} + 2Z_{\alpha\mu}F_{\beta\nu}F^{\alpha\beta}))] \quad (15)
\end{aligned}$$

where $c_w = \cos \theta_W$, $s_W = \sin \theta_W$. $G_{\mu\nu}$, $F_{\mu\nu}$, $Z_{\mu\nu} = \partial_\mu Z_\nu - \partial_\nu Z_\mu$ are the field strengths for the gluon, photon and Z particles, respectively. Note that the above interactions are obtained at the unification scale where $g_Y = \sqrt{3/5}g^U$ and $\sin^2 \theta_W = 3/8$, and $g^C = g^L = g^R = g^U$.

From the above we see that the triple neutral gauge boson interactions are uniquely determined unlike the case with SM gauge group studied in Ref. [8]. These interactions are also different from those predicted by $SU(5)$ model [9]. This can be used to test the model [11].

The fermion-gauge boson interactions can readily be obtained by expanding eq. (14). The Yukawa coupling terms and Higgs potential terms can also be obtained using results in eqs. (6) and (13).

We have constructed a NCQFT unification model of strong and electroweak interactions based on $SU(3)_C \times SU(3)_L \times SU(3)_R \times Z_3$ group. In this model all interactions are determined. New gauge boson self- and fermion-gauge interactions are predicted. If the noncommutative scale turns out to be low, the model can be tested experimentally. This model provides an example which can consistently describe the strong and electroweak in-

teractions and their unification, and allows a systematic investigation of the hypothesis of noncommutative space-time.

Before closing we would like to make two comments on some possible extensions of the model discussed here. One of them concerns about E_6 extension of the model. The $SU(3)_C \times SU(3)_L \times SU(3)_R$ group can be embedded into E_6 group. One therefore can try to construct a NCQFT based on E_6 . With this group the gauge bosons are in the 78 adjoint representation and the fermions and Higgses are in the 27 fundamental representations [12]. A NCQFT model can be constructed following the procedures discussed earlier. This model is very similar to the trinification model with the advantage that no additional Z_3 symmetry is needed. There are however differences and complications. In addition to the gauge bosons in the trinification model, there are also 54 colored gauge bosons. These particles mediate proton decays. Therefore they have to be made heavy. To achieve this more Higgs representations will have to be introduced which complicate the theory [12]. The trinification model is simpler in terms of particle contents.

The other comment concerns another approach to construct trinification model with noncommutative space-time without the use of Seiberg-Witten mapping adopted in Ref. [13]. In this approach one first constructs fields in $U(N)$ product groups and then break the symmetry spontaneously to $SU(N)$ product group. For the trinification model, one can extend the group to $U(3)_C \times U(3)_L \times U(3)_R$. The gauge field representation is

$$27 = (9, 1, 1) + (1, 9, 1) + (1, 1, 9).$$

In Ref. [13] the symmetry breaking of $U(N)$ to $SU(N)$ is assumed to be achieved by non-zero VEV's of representations S_i which transform as singlets under the $SU(N)$ but with non-zero charge for $U(1)$ subgroup of $U(N)$. Following Ref. [13] one can introduce three S_i fields for each of the factor $U(N)$ group. The VEV's of these fields break the group to the trinification group discussed earlier producing three heavy gauge bosons. However it has been shown that models based on such approach violates unitarity [17]. This approach may not lead to a realistic model.

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Note Added: Another consistent noncommutative grand unified model based on $SO(10)$ has been constructed by P. Aschieri et al. (hep-th/0205214), three months after this work.

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